Sensitivity and Errors

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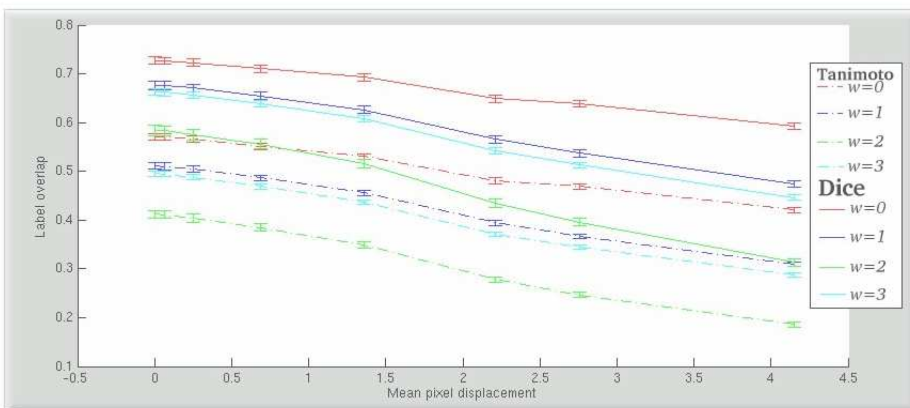
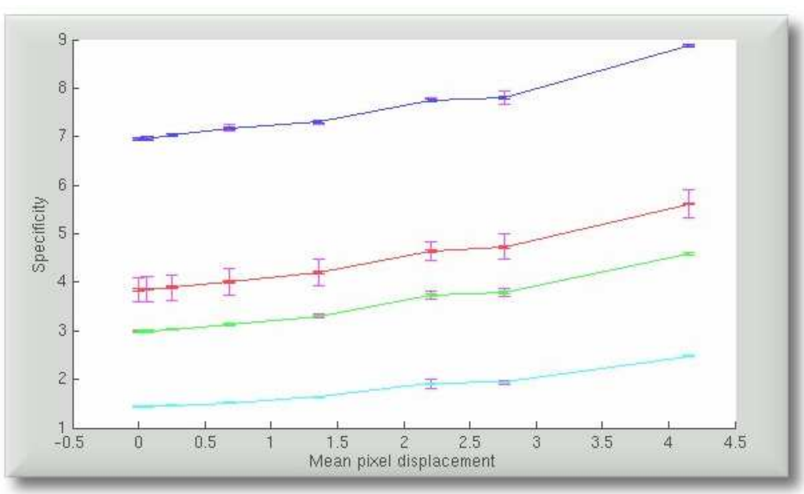
Abstract

This document assembles a collection of notes on how one should calculate the sensitivity of measures such as model Specificity and Generalisability, as well as label overlap. Derivation of error bars is included as well, albeit it is work in progress.

1 Background

EXAMPLE plots are included in this section, which attempts to illustrate the general aims and add some context. We are investigating plots which depict a certain measure m versus the extent of mean pixel displacement. These plots typically reflect on some measure of 'goodness' as data is degraded. Mean pixel displacement is closely-related to the degree of *mis-registration* in the set of images under investigation. Below are 2 blended figures which practically visualise that.





It is worth noticing that all curves are sampled at 8 separate points. Each such point corresponds to one particular value of mean pixel displacement. The value measured, namely m, is calculated over 10 instantiations of an image set, from which the average has been derived.

We are interested in two separate 'sources' of error (uncertainty):

1. **Error that is associated with the instantiation process.** As the number of instantiation is finite, our measures are susceptible to some fluctuation. We absolutely must account for bias, which is due to the random instantiation process.
2. **Error that is associated with the calculation of the value** m. To obtain the values which we seek, a set of synthetic images is generated. Since that set is limited in term of its size, corresponding error bars need to be bound.

2 Combining the Errors

To get meaningful plots, where error bars faithfully reflect on truth, we ought to better understand our error sources. They appear to be independent, but this observation does not simplify matters.

Let us look at the plots vertically, taking one value of mean pixel displacement at the time. Each value of displacement, donated by d, is produced by warping a set of aligned images, i.e. images where there is no inherent displacement.

Let us define σ_{mi} to be the predicted error in the estimate of m for a warp instance $i : (1..N)$. We can then obtain the mean

$$\overline{\sigma_m} = \sum_i \frac{\sigma_{mi}}{N} \tag{1}$$

and the standard error is thus

$$SE_{\sigma_m} = \frac{SD(\sigma_{mi})}{\sqrt{N-1}}. \tag{2}$$

As for the mean of the measurement m for the given displacement value m,

$$\overline{m_d} = \sum_i m_i/N \tag{3}$$

and the corresponding standard error

$$SE_{\overline{m}} = \frac{SD(m_i)}{\sqrt{N-1}}. \tag{4}$$

3 Sensitivity

Sensitivity allows us to reason about the ability of a given measure to discern, by means of simple calculation, one magnitude displacement from another. The value of sensitivity is affected by uncertainty as well – uncertainty which must be propagated from formulations in §2.

We introduce another index j to avoid confusion with i, which re-appears once the formulae are expanded. The sensitivity at a point j in our new sensitivity plot should be computed as follows:

$$Sens_j = \frac{(\overline{m_j} - \overline{m_0})}{d_j} / (\overline{\sigma_m})_j = \frac{TOP}{BOTT} \tag{5}$$

4 Error in Sensitivity

From the above, the corresponding errors can now be derived:

$$\sigma_{TOP}^2 = (SE_{\overline{m}})_j^2 + (SE_{\overline{m}})_0^2 \tag{6}$$

$$\sigma_{BOTT}^2 = (SE_{\sigma_m})_j^2 \tag{7}$$

$$\sigma_{Sens_j}^2 = F(\sigma_{TOP}^2, \sigma_{BOTT}^2) \tag{8}$$

The function which (8) should become is yet undecided.