Proposal for Perturbation Methodology
List of Criteria

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Our aim is to apply properly controlled (in terms of mean displacement) perturbations to arbitrary sets of images. Images are expected to be of the brain which occupies the centre of the images. Most preferably, perturbation should be homogeneous in order for images to be treated without any bias. Perturbation must affect all regions of a given image and abstain from using knowledge about objects which the image depicts.

Formulation of the Problem

Let us assume that we were given a set of n images $\mathbf{I}_1, \mathbf{I}_2, .., \mathbf{I}_n$. Each of these images is two-dimensional – in the simpler case at least. An image is N pixels nigh and M pixels wide. Each perturbation (displacement) Δ has a direction $\overrightarrow{\Lambda}$ and an intensity (vector length) associated with it. Let us define the total displacement to be Δ_{total} and the average $\Delta_{average}$ accordingly. Then,

$$\Delta_{total} = \sum_{k=1}^{n} \sum_{i=1}^{N} \sum_{i=1}^{M} \Delta_{k_{i,j}} \quad (1)$$

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and

$$\Delta_{average} = \frac{\Delta_{total}}{nMN}. \tag{2}$$

Having formulated it in this way, a more proper average should treat the displacements in each image separately and not aggregate displacements in each of the n images.

$$\mathbf{I}'_{k_{i,j}} = \sum_{k=1}^{n} \sum_{i=1}^{N} \sum_{i=1}^{M} \mathbf{I_{k_{i,j}}} + \Delta_{k_{i,j}} \tag{3}$$

only *reflects* on how the new set of images are generated. Rather than calculating a sum, it forms a set of matrices (in a vector-wise assignment).

Some displacement $\Delta_{k_{i,j}}$ has been applied to each of the pixels, $\mathbf{I_{k_{i,j}}}$, in each of of the images in the set. We seek a way of selecting Δ in a way which obeys certain rules. The goal is the obtain a stack whose members are the images $\mathbf{I}_1, \mathbf{I}_2, .., \mathbf{I}_n$ and where each member of the stack has an increasing amount of displacement applied. The perturbation method needs to sensibly pick values for $\Delta_{k_{i,j}}$ so that a clear relationship among stack member should emerge.

Proposal of Particular Criteria

1. **Range of scales.** Images are subjected to warps of varying intensity. We seek a perturbation framework that makes this intensity (magnitude) trivial to increase by a fixed and known amount. It should be flexible enough in terms of the scales supported or else obscenely large deformations cannot be investigated. One of the interesting properties to investigate is: *at which point does the amount of perturbation become to difficult to detect and quantify?*

2. **Diffeomorphism.** No folding, tearing, etc.

3. **Invertibility.** This is a useful property if one wishes to recover the images from their perturbed version. Being invertible is not, however, a much-required trait.

4. **Large number of perturbation 'sources'.** To make the effects of perturbation less local and more global, a large number of random processes, e.g. warps, need to be spread within the image boundaries.

5. **No pixel condensation at images edges and corners.** This 'stuffing' of pixels tends to happen when there is not sufficient freedom for pixels to be moved outside the image boundaries.

6. **Stochastic.** Perturbation needs to posses a stochastic nature. Points needs to be displaced by a random unit, which is drawn from a normal distribution

7. **Predictable $E[\Delta_{average}]$.** For any given point, the distribution of its displacements must be well-understood.

8. **Similar distributions across the entire image.** One would hope that displacements affect all parts of the image similarly. This may be difficult to assure.

9. **Perturbation scales that increase in a well-behaved manner.** $E[\Delta_{average}]$ should increase/decrease monotonically and also linearly or logarithmically (any other predictable curve which allows fitting should do) as function of the perturbation scale. If it increases too rapidly, valuable data might get neglected.

10. **No re-re-sampling error.** When images are warped (transformed), interpolated and re-sampled, there is a certain loss of detail, often visible in the form of blurring. There is an error associated with it too. Good perturbation will avoid errors (striving to reach 0) as errors add noise to the results.

Possible Perturbation Mechanisms

Current Perturbation Methods

A fixed number of clamped-plate splines (CPS) are placed at a corresponding set of fixed positions. Their direction is chosen stochastically and their intensity (magnitude) is increased to achieve varying levels of perturbations. Linear relationship was assumed between the intensity and the amount of perturbation, i.e. the average number of pixels by which a point is displaced. (For more details on the actual implementation, ask Vlad. Carole says that it is aggregation of warps that increases the perturbation. So, added displacement is only known vaguely)

The expected pitfalls are as follows:

- The composition of warps was not taken into consideration. Increasing warp intensities do not necessarily increase the displacements (not linearly at least).

- Boundaries prevented warps from being applied properly. In the case of brain images, this may not be crucial.

- Images began to break some constraints when the perturbation increased excessively.

Image Data

The images are typically *255 x 255* pixels in size. Some datasets contain smaller images, but they all are images of the brain. Some images contain a great deal of uniform black/greys surrounding the brain, whereas some do not.

Ways to Proceed

There are certain behaviours that need to be looked at systematically:

- Investigate the warp fields for insight into displacements in terms of number of pixels.
- Learn the re-sampling errors (if any) using synthetic data.
- Study the rate at which displacement (in terms of pixels) increases for a given perturbation scheme.
- Learn how distributions vary across the image regions.

What follows are suggestions, which can improve the current scheme for perturbing the data:

- Reduce the size of warps and use a larger number of them.
- Define a spatial constraint which the centres of warps are subjected to.
- Extend the image boundaries and fill these with appropriate grey-level values. This will cater for more freedom of pixel movement.
- (Carole) Warps need to be aggregated. In other words, if perturbation is based on warps, then at each stage one has to add new warp/s atop the existing ones – the ones from previous perturbation steps.